nag_bessel_i0 (s18aec)

1.   Purpose

nag_bessel_i0 (s18aec) returns the value of the modified Bessel function $I_0(x)$.

2.   Specification

```
#include <nag.h>
#include <nags.h>

double nag_bessel_i0(double x, NagError *fail)
```

3.   Description

This function evaluates an approximation to the modified Bessel function of the first kind, $I_0(x)$.

The function is based on Chebyshev expansions.

For large x, the function must fail because of the danger of overflow in calculating e^x.

4.   Parameters

x

> Input: the argument x of the function.

fail

> The NAG error parameter, see the Essential Introduction to the NAG C Library.

5.   Error Indications and Warnings

NE_REAL_ARG_GT

> On entry, $|\mathbf{x}|$ must not be greater than $\langle value \rangle$: $\mathbf{x} = \langle value \rangle$.
>
> $|\mathbf{x}|$ is too large and the function returns the approximate value of $I_0(x)$ at the nearest valid argument.

6.   Further Comments

6.1.   Accuracy

Let δ and ϵ be the relative errors in the argument and result respectively.

If δ is somewhat larger than the **machine precision** (i.e., if δ is due to data errors etc.), then ϵ and δ are approximately related by $\epsilon \simeq |xI_1(x)/I_0(x)| \, \delta$.

However, if δ is of the same order as **machine precision**, then rounding errors could make ϵ slightly larger than the above relation predicts.

For small x the amplification factor is approximately $x^2/2$, which implies strong attenuation of the error, but in general ϵ can never be less than the **machine precision**.

For large x, $\epsilon \simeq x\delta$ and we have strong amplification of errors. However, the function must fail for quite moderate values of x, because $I_0(x)$ would overflow; hence in practice the loss of accuracy for large x is not excessive. Note that for large x the errors will be dominated by those of the **math library** function exp.

6.2.   References

Abramowitz M and Stegun I A (1968) *Handbook of Mathematical Functions* Dover Publications, New York ch 9 p 374.

7.   See Also

nag_bessel_i1 (s18afc)

8.   Example

The following program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

8.1.   Program Text

```
/* nag_bessel_i0(s18aec) Example Program
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

main()
{
  double x, y;

  /* Skip heading in data file */
  Vscanf("%*[^\n]");
  Vprintf("s18aec Example Program Results\n");
  Vprintf("      x           y\n");
  while (scanf("%lf", &x) != EOF)
    {
      y = s18aec(x, NAGERR_DEFAULT);
      Vprintf("%12.3e%12.3e\n", x, y);
    }
  exit(EXIT_SUCCESS);
}
```

8.2.   Program Data

```
s18aec Example Program Data
            0.0
            0.5
            1.0
            3.0
            6.0
            8.0
            10.0
            15.0
            20.0
            -1.0
```

8.3.   Program Results

```
s18aec Example Program Results
      x           y
  0.000e+00   1.000e+00
  5.000e-01   1.063e+00
  1.000e+00   1.266e+00
  3.000e+00   4.881e+00
  6.000e+00   6.723e+01
  8.000e+00   4.276e+02
  1.000e+01   2.816e+03
  1.500e+01   3.396e+05
  2.000e+01   4.356e+07
 -1.000e+00   1.266e+00
```